MCM

Monday, April 9, 2018 5:28 PM

Matrix Chain Multiplication

Given List of Matrices

$A_1, A_2, A_3, \cdots, A_n$

Task is to compute the minimum number of operations for determining their product.

Matrix example

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

rows ↓ columns ↓

A is 3×2

$$B = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$

B is 2×4

A B ⟶ C

$(3 \times \boxed{2 = 2} \times 4)$ ✓

C is 3×4

B A

$2 \times \boxed{\#\quad 3} \times 2$

$\times$

$4 \times \boxed{2 \qquad 2} \times 3$

↑ ↑ ↑

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$4 \times 3 \times 2 = 12$

$$B = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$

[1 1 2 4]

0·3 + 1·1   3·1 + 0·1

$$C = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 3 & 2 \end{bmatrix}$$

Number of operations

Row(A) · Cols(B) (Row(B) or Cols(A))

4 1 1 3 3 2
A B C

(A B) C   A(B C)

4×1×3 + 4×3×2   1×3×2 + 4×1×2

36   14

$A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot \ldots \cdot A_n$

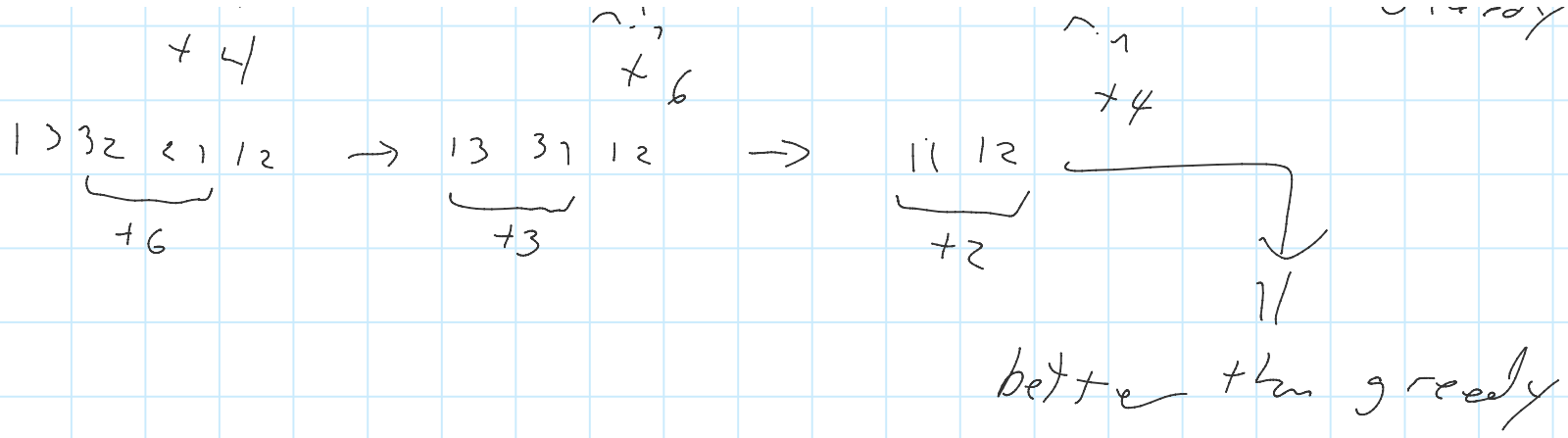
we do NOT move Matrices

Greedy idea
find the cheapest pair
and do the multiply

1 3 3 2 2 1 1 2 → 1 3 3 2 2 2 → 1 2 2 2 →
min +4   min +6   min +4   Greedy = 14

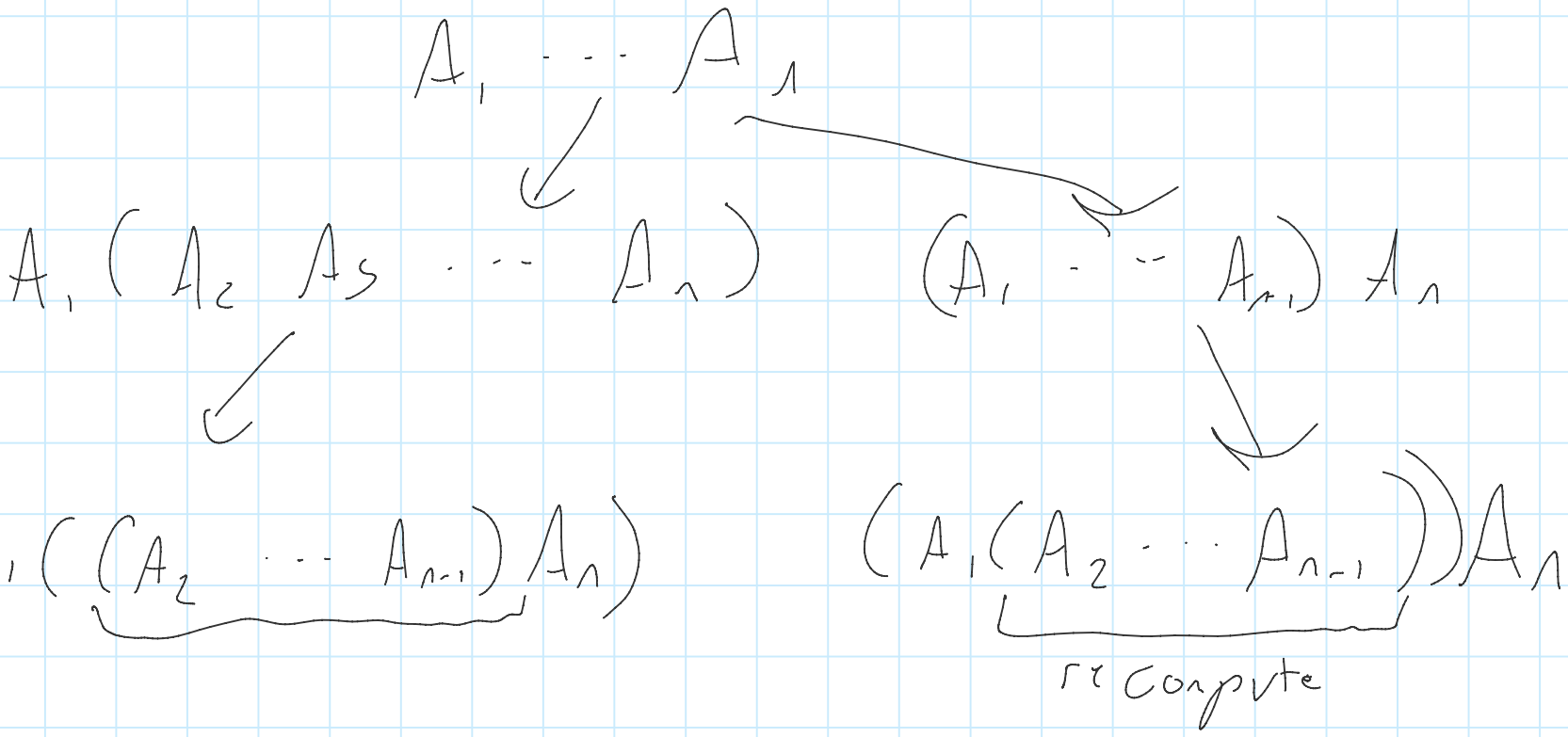
but

but consider



Dynamic Programming

$A_1 \cdot A_2 \cdot A_3 \cdots A_n$



Goal:

Runtime $O(N^3)$

State $O(N^2)$

runtime for our states $O(N)$